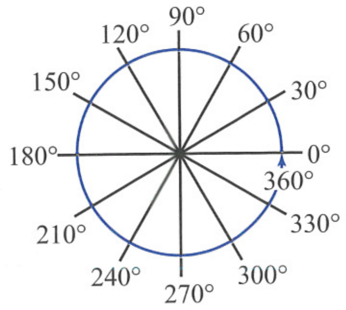
(2) More commonly, one uses a small unit called a **degree**. For this, we divide the circle into 360 equal arcs; the angle corresponding to each small arc is 1 degree.

> One complete turn is 360 degrees (denoted 360°).

Consequently,



- A straight angle is half of a complete turn, so is 180°.
- A right angle is a quarter of a complete turn, so is 90°.

Notice that the word "of" in these sentences is one of the interpretations of fraction multiplication. Consequently, students often learn to convert fractions of a circle into degrees in the same grade that they learn to multiply fractions.

EXAMPLE 4.1. *How many degrees are in $\frac{1}{3}$ of a complete turn? In $\frac{1}{5}$ of a complete turn? How many degrees are in $\frac{1}{6}$ of a right angle?*

Solution:

$$\left.\begin{array}{lclcl} \frac{1}{3}\text{ turn} &=& \frac{1}{3}\times 360^\circ &=& \left(\frac{1}{3}\times 36\right)\times 10^\circ \;=\; 120^\circ \\ \frac{1}{5}\text{ turn} &=& \frac{1}{5}\times 360^\circ &=& \left(\frac{1}{10}\times 360^\circ\right)\times 2 \;=\; 72^\circ \\ \frac{1}{6}\text{ right angle} &=& \frac{1}{6}\times 90^\circ &=& 15^\circ. \end{array}\right\}\quad \text{by mental math!}$$

One degree is small enough that all angles are closely approximated by a whole number of degrees. Consequently, fractions are unnecessary for most purposes — all angle measurements and calculations involve only whole numbers.

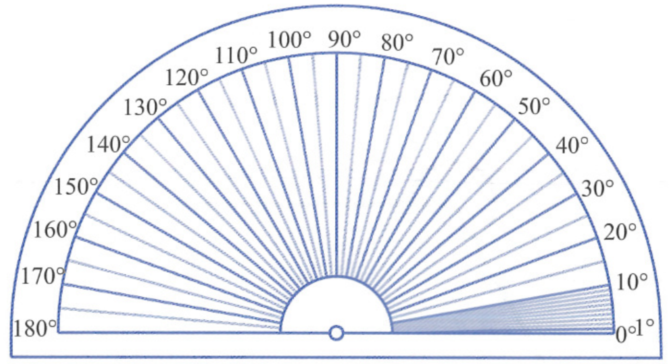
Why 360°?

But why use the number 360? The unit 'degree' originated with the Mesopotamians (circa 4000 B.C.), who based their entire number system on the number 60. Some modern measurements retain vestiges of the Mesopotamian system: an hour is 60 minutes, a minute is 60 seconds, and a circle has $6 \times 60 = 360$ degrees. The use of 60 is a way of avoiding fractions: when a unit is made of 60 equal parts, many fractional units can be expressed as whole numbers. For example, $\frac{1}{3}$ of 60 is 20, $\frac{1}{4}$ of 60 is 15, $\frac{1}{5}$ of 60 is 12, and $\frac{1}{6}$ is 10.

In particular, dividing a circle into 360 equal parts ensures that common angles are expressed in degrees as whole numbers, including the angles in Example 4.1. Calculations with common angles become much easier. For example, it is much easier to add $30° + 45°$ than to add $\frac{1}{12}$ turn $+\ \frac{1}{8}$ turn. It also makes angles much easier to *teach* because children can learn to measure and calculate with angles in degrees before they have mastered fractions.

Learning To Use a Protractor

Degrees are introduced in Primary Math 4A. At that point, students learn to use a protractor to measure angles and to draw angles of specified sizes.



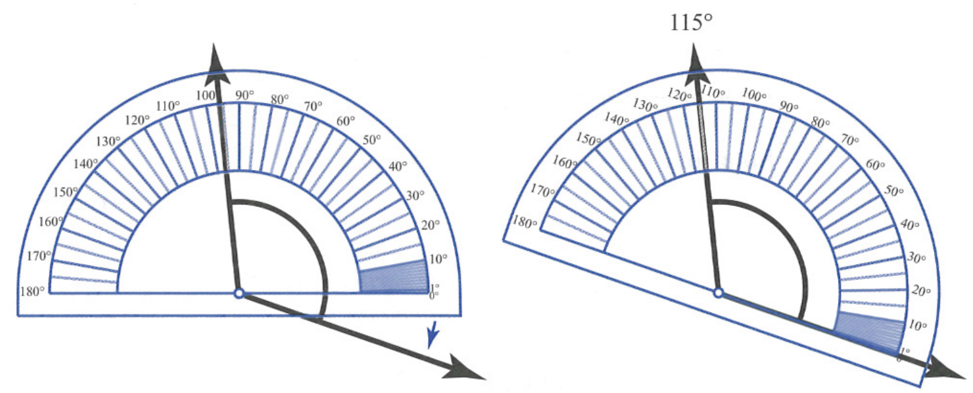
Looking at a protractor, one can see how its construction relates to the definition of a degree. One can envision starting with a blank semicircle of cardboard or plastic, wrapping a flexible tape measure along its outer edge, and drawing 180 equally-spaced marks. The angle formed by rays from the center point through two consecutive marks is 1°.

This angle is 1 degree.

1 degree is a surprisingly small angle! It is roughly the width of a pencil held at arm's length from your eye. Consequently, *one can describe the size of an angle very accurately by measuring it to the nearest degree.*

Measuring angles with a protractor is very much like checking for right angles, which students learned to do in grade 3. There are three steps:

1. Align the center hole on the vertex of the angle.
2. Align 0° with one side of the angle.
3. Read the measure of the angle on the appropriate scale (most protractors have two scales).



Careful instruction is required here! It is common for students to make errors on all three of these steps. You will learn about these errors in Homework Problem 10.