CONTENTS

LIST OF FORMULAE AND STATISTICAL TABLES (MF15)

PURE MATHEMATICS

Algebraic series

Binomial expansion:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots + b^n,$$ where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Maclaurin's expansion:

$$f(x) = f(0) + x\,f'(0) + \frac{x^2}{2!}f''(0) + \ldots + \frac{x^n}{n!}f^{(n)}(0) + \ldots$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x|<1)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^r}{r!} + \ldots \qquad (\text{all } x)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + \ldots \qquad (\text{all } x)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^r x^{2r}}{(2r)!} + \ldots \qquad (\text{all } x)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + \frac{(-1)^{r+1}x^r}{r} + \ldots \qquad (-1 < x \le 1)$$

Partial fractions decomposition

Non-repeated linear factors:

$$\frac{px+q}{(ax+b)(cx+d)} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)}$$

Repeated linear factors:

$$\frac{px^2+qx+r}{(ax+b)(cx+d)^2} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)} + \frac{C}{(cx+d)^2}$$

Non-repeated quadratic factor:

$$\frac{px^2+qx+r}{(ax+b)(x^2+c^2)} = \frac{A}{(ax+b)} + \frac{Bx+C}{(x^2+c^2)}$$

Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A \equiv 2 \sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin P + \sin Q \equiv 2 \sin \tfrac{1}{2}(P+Q) \cos \tfrac{1}{2}(P-Q)$$

$$\sin P - \sin Q \equiv 2 \cos \tfrac{1}{2}(P+Q) \sin \tfrac{1}{2}(P-Q)$$

$$\cos P + \cos Q \equiv 2 \cos \tfrac{1}{2}(P+Q) \cos \tfrac{1}{2}(P-Q)$$

$$\cos P - \cos Q \equiv -2 \sin \tfrac{1}{2}(P+Q) \sin \tfrac{1}{2}(P-Q)$$

Principal values:

$$-\tfrac{1}{2}\pi \le \sin^{-1} x \le \tfrac{1}{2}\pi \qquad (|x| \le 1)$$

$$0 \le \cos^{-1} x \le \pi \qquad (|x| \le 1)$$

$$-\tfrac{1}{2}\pi < \tan^{-1} x < \tfrac{1}{2}\pi$$

Derivatives

f(x)	f′(x)
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\sec x$	$\sec x \tan x$

Integrals

(Arbitrary constants are omitted; a denotes a positive constant.)

f(x)	$\int$ f(x) dx	
$\frac{1}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)$	$(\lvert x\rvert < a)$
$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right)$	$(x > a)$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right)$	$(\lvert x\rvert < a)$
$\tan x$	$\ln(\sec x)$	$(\lvert x\rvert < \frac{1}{2}\pi)$
$\cot x$	$\ln(\sin x)$	$(0 < x < \pi)$
$\operatorname{cosec} x$	$-\ln(\operatorname{cosec} x + \cot x)$	$(0 < x < \pi)$
$\sec x$	$\ln(\sec x + \tan x)$	$(\lvert x\rvert < \frac{1}{2}\pi)$

Vectors

The point dividing AB in the ratio $\lambda : \mu$ has position vector $\frac{\mu\mathbf{a} + \lambda\mathbf{b}}{\lambda + \mu}$

If A is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector $\mathbf{b}$ is given by $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through A with direction vector $\mathbf{b}$ has cartesian equation

$$\frac{x-a_1}{b_1} = \frac{y-a_2}{b_2} = \frac{z-a_3}{b_3} \ (= \lambda)$$

The plane through A with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation

$$n_1 x + n_2 y + n_3 z + d = 0 \text{ where } d = -\mathbf{a.n}$$

Numerical methods

Euler's Method with step size h:

$$y_{n+1} = y_n + h\,\mathrm{f}(x_n, y_n)$$

Improved Euler Method with step size h:

$$u_{n+1} = y_n + h\,\mathrm{f}(x_n, y_n)$$

$$y_{n+1} = y_n + \frac{h}{2}\left[\mathrm{f}(x_n, y_n) + \mathrm{f}(x_{n+1}, u_{n+1})\right]$$

STATISTICS

Standard discrete distributions

Distribution of X	$\mathrm{P}(X = x)$	Mean	Variance
Binomial $\mathrm{B}(n,p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Poisson $\mathrm{Po}(\lambda)$	$e^{-\lambda}\frac{\lambda^x}{x!}$	λ	λ

Sampling and testing

Unbiased variance estimate from a single sample:

$$s^2 = \frac{1}{n-1}\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right) = \frac{1}{n-1}\Sigma(x-\bar{x})^2$$

Regression and correlation

Estimated product moment correlation coefficient:

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\left\{\Sigma(x-\bar{x})^2\right\}\left\{\Sigma(y-\bar{y})^2\right\}}} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)\left(\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right)}}$$

Estimated regression line of y on x:

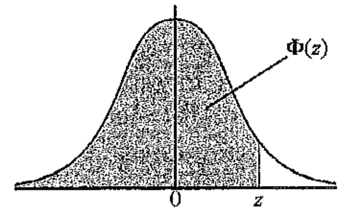
$$y - \bar{y} = b(x - \bar{x}), \quad \text{where } b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1 then, for each value of z, the table gives the value of $\Phi(z)$, where

$$\Phi(z) = \mathrm{P}(Z \le z).$$

For negative values of z use $\Phi(-z) = 1 - \Phi(z)$.



z	0	1	2	3	4	5	6	7	8	9	ADD 1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that

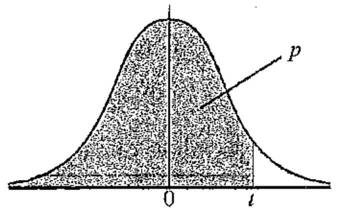
$$\mathrm{P}(Z \le z) = p.$$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE t-DISTRIBUTION

If T has a t-distribution with ν degrees of freedom then, for each pair of values of p and ν, the table gives the value of t such that

$$P(T \le t) = p.$$



p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
ν = 1	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.894	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.689
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.660
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level
Higher 2

MATHEMATICS 9740/01

Paper 1

October/November 2014

3 hours

Additional Materials: Answer Paper
Graph paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

1 The function f is defined by

$$f : x \mapsto \frac{1}{1-x}, \quad x \in \mathbb{R},\ x \neq 1,\ x \neq 0.$$

(i) Show that $f^2(x) = f^{-1}(x)$. [4]

(ii) Find $f^3(x)$ in simplified form. [1]

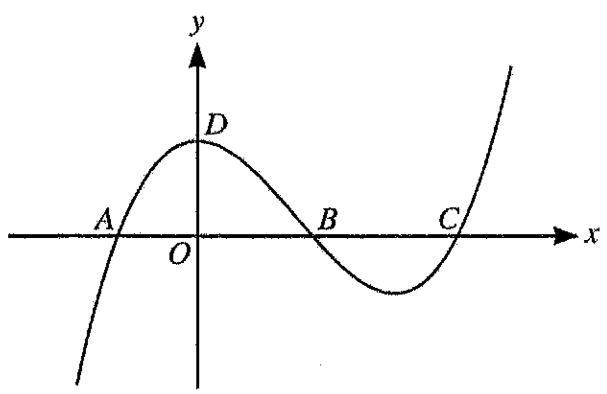
2 The curve C has equation $x^2y + xy^2 + 54 = 0$. Without using a calculator, find the coordinates of the point on C at which the gradient is -1, showing that there is only one such point. [6]

3 (i) Given that $\mathbf{a} \times \mathbf{b} = \mathbf{0}$, what can be deduced about the vectors $\mathbf{a}$ and $\mathbf{b}$? [2]

(ii) Find a unit vector $\mathbf{n}$ such that $\mathbf{n} \times (\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) = \mathbf{0}$. [2]

(iii) Find the cosine of the acute angle between $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and the z-axis. [1]

4



The diagram shows the curve $y = f(x)$. The curve crosses the x-axis at the points A, B and C, and has a maximum turning point at D where it crosses the y-axis. The coordinates of A, B, C and D are $(-a, 0)$, $(b, 0)$, $(c, 0)$ and $(0, d)$ respectively, where a, b, c and d are positive constants.

(i) Sketch the curve $y^2 = f(x)$, stating, in terms of a, b, c and d, the coordinates of any turning points and of the points where the curve crosses the x-axis. [4]

(ii) What can be said about the tangents to the curve $y^2 = f(x)$ at the points where it crosses the x-axis? [1]

5 It is given that $z = 1 + 2i$.

(i) Without using a calculator, find the values of z^2 and $\frac{1}{z^3}$ in cartesian form $x + iy$, showing your working. [4]

(ii) The real numbers p and q are such that $pz^2 + \frac{q}{z^3}$ is real. Find, in terms of p, the value of q and the value of $pz^2 + \frac{q}{z^3}$. [3]

6 (a) A sequence $p_1, p_2, p_3, \ldots$ is given by

$$p_1 = 1 \quad \text{and} \quad p_{n+1} = 4p_n - 7 \text{ for } n \geqslant 1.$$

(i) Use the method of mathematical induction to prove that

$$p_n = \tfrac{1}{3}(7 - 4^n).$$ [5]

(ii) Find $\sum_{r=1}^{n} p_r$. [3]

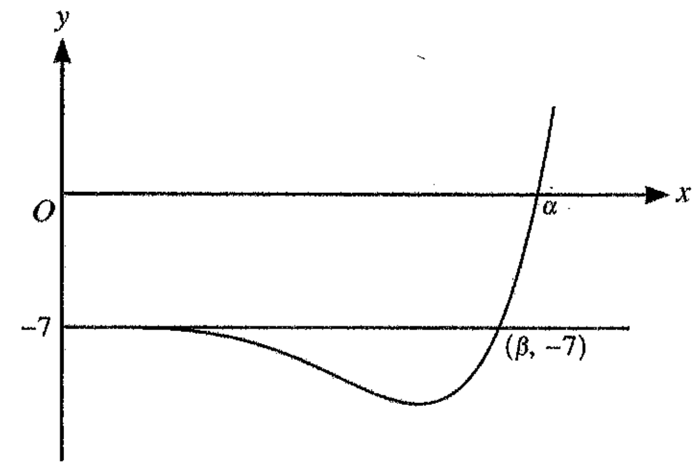
(b) The sum, S_n, of the first n terms of a sequence $u_1, u_2, u_3, \ldots$ is given by

$$S_n = 1 - \frac{1}{(n+1)!}.$$

(i) Give a reason why the series Σu_r converges, and write down the value of the sum to infinity. [2]

(ii) Find a formula for u_n in simplified form. [2]

7



It is given that $f(x) = x^6 - 3x^4 - 7$. The diagram shows the curve with equation $y = f(x)$ and the line with equation $y = -7$, for $x \geqslant 0$. The curve crosses the positive x-axis at $x = \alpha$, and the curve and the line meet where $x = 0$ and $x = \beta$.

(i) Find the value of α, giving your answer correct to 3 decimal places, and find the exact value of β. [2]

(ii) Evaluate $\int_{\beta}^{\alpha} f(x)\,dx$, giving your answer correct to 3 decimal places. [2]

(iii) Find, in terms of $\sqrt{3}$, the area of the finite region bounded by the curve and the line, for $x \geqslant 0$. [3]

(iv) Show that $f(x) = f(-x)$. What can be said about the six roots of the equation $f(x) = 0$? [4]

8 It is given that $f(x) = \dfrac{1}{\sqrt{(9 - x^2)}}$, where $-3 < x < 3$.

(i) Write down $\int f(x)\,dx$. [1]

(ii) Find the binomial expansion for $f(x)$, up to and including the term in x^6. Give the coefficients as exact fractions in their simplest form. [4]

(iii) Hence, or otherwise, find the first four non-zero terms of the Maclaurin series for $\sin^{-1}(\frac{1}{3}x)$. Give the coefficients as exact fractions in their simplest form. [4]

9 Planes p and q are perpendicular. Plane p has equation $x + 2y - 3z = 12$. Plane q contains the line l with equation $\dfrac{x-1}{2} = \dfrac{y+1}{-1} = \dfrac{z-3}{4}$. The point A on l has coordinates $(1, -1, 3)$.

(i) Find a cartesian equation of q. [4]

(ii) Find a vector equation of the line m where p and q meet. [4]

(iii) B is a general point on m. Find an expression for the square of the distance AB. Hence, or otherwise, find the coordinates of the point on m which is nearest to A. [5]

10 The mass, x grams, of a certain substance present in a chemical reaction at time t minutes satisfies the differential equation

$$\frac{dx}{dt} = k(1 + x - x^2),$$

where $0 \leqslant x \leqslant \frac{1}{2}$ and k is a constant. It is given that $x = \frac{1}{2}$ and $\dfrac{dx}{dt} = -\frac{1}{4}$ when $t = 0$.

(i) Show that $k = -\frac{1}{5}$. [1]

(ii) By first expressing $1 + x - x^2$ in completed square form, find t in terms of x. [5]

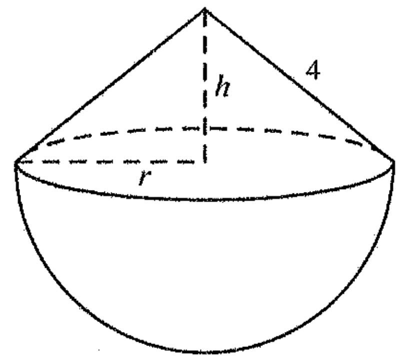
(iii) Hence find

(a) the exact time taken for the mass of the substance present in the chemical reaction to become half of its initial value, [1]

(b) the time taken for there to be none of the substance present in the chemical reaction, giving your answer correct to 3 decimal places. [1]

(iv) Express the solution of the differential equation in the form $x = f(t)$ and sketch the part of the curve with this equation which is relevant in this context. [5]

11 [It is given that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$ and that the volume of a circular cone with base radius r and height h is $\frac{1}{3}\pi r^2 h$.]



A toy manufacturer makes a toy which consists of a hemisphere of radius r cm joined to a circular cone of base radius r cm and height h cm (see diagram). The manufacturer determines that the length of the slant edge of the cone must be 4 cm and that the total volume of the toy, $V\,\text{cm}^3$, should be as large as possible.

(i) Find a formula for V in terms of r. Given that $r = r_1$ is the value of r which gives the maximum value of V, show that r_1 satisfies the equation $45r^4 - 768r^2 + 1024 = 0$. [6]

(ii) Find the two solutions to the equation in part **(i)** for which $r > 0$, giving your answers correct to 3 decimal places. [2]

(iii) Show that one of the solutions found in part **(ii)** does not give a stationary value of V. Hence write down the value of r_1 and find the corresponding value of h. [3]

(iv) Sketch the graph showing the volume of the toy as the radius of the hemisphere varies. [3]

MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level
Higher 2

MATHEMATICS

9740/02

Paper 2

October/November 2014

3 hours

Additional Materials: Answer Paper
Graph paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 A curve C has parametric equations

$$x = 3t^2, \quad y = 6t.$$

(i) Find the value of t at the point on C where the tangent has gradient 0.4. [3]

(ii) The tangent at the point $P\,(3p^2, 6p)$ on C meets the y-axis at the point D. Find the cartesian equation of the locus of the mid-point of PD as p varies. [4]

2 Using partial fractions, find

$$\int_0^2 \frac{9x^2 + x - 13}{(2x-5)(x^2+9)}\,\mathrm{d}x.$$

Give your answer in the form $a \ln b + c \tan^{-1} d$, where a, b, c and d are rational numbers to be determined. [9]

3 In a training exercise, athletes run from a starting point O to and from a series of points, A_1, A_2, A_3, … , increasingly far away in a straight line. In the exercise, athletes start at O and run stage 1 from O to A_1 and back to O, then stage 2 from O to A_2 and back to O, and so on.

(i)

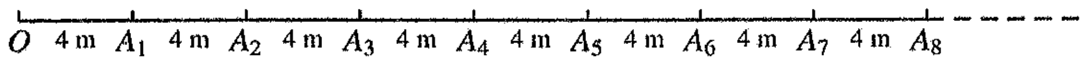


Fig. 1

In Version 1 of the exercise, the distances between adjacent points are all 4 m (see Fig. 1).

(a) Find the distance run by an athlete who completes the first 10 stages of Version 1 of the exercise. [2]

(b) Write down an expression for the distance run by an athlete who completes n stages of Version 1. Hence find the least number of stages that the athlete needs to complete to run at least 5 km. [4]

(ii)

Fig. 2

In Version 2 of the exercise, the distances between the points are such that $OA_1 = 4$ m, $A_1A_2 = 4$ m, $A_2A_3 = 8$ m and $A_nA_{n+1} = 2A_{n-1}A_n$ (see Fig. 2). Write down an expression for the distance run by an athlete who completes n stages of Version 2. Hence find the distance from O, and the direction of travel, of the athlete after he has run exactly 10 km using Version 2. [5]

4 **(a)** The complex number z satisfies $|z + 5 - \mathrm{i}| = 4$.

(i) On an Argand diagram show the locus of z. [2]

(ii) The complex number z also satisfies $|z - 6\mathrm{i}| = |z + 10 + 4\mathrm{i}|$. Find exactly the possible values of z, giving your answers in the form $x + \mathrm{i}y$. [4]

(b) It is given that $w = (\sqrt{3}) - \mathrm{i}$.

(i) Without using a calculator, find an exact expression for w^6. Give your answer in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $0 \leqslant \theta < 2\pi$. [3]

(ii) Without using a calculator, find the three smallest positive whole number values of n for which $\frac{w^n}{w^*}$ is a real number. [4]

Section B: Statistics [60 marks]

5 An internet retailer has compiled a list of 10 000 regular customers and wishes to carry out a survey of customer opinions involving 5% of its customers.

(i) Describe how the marketing manager could choose customers for this survey using systematic sampling. [2]

(ii) Give one advantage and one disadvantage of systematic sampling in this context. [2]

6 A team in a particular sport consists of 1 goalkeeper, 4 defenders, 2 midfielders and 4 attackers. A certain club has 3 goalkeepers, 8 defenders, 5 midfielders and 6 attackers.

(i) How many different teams can be formed by the club? [2]

One of the midfielders in the club is the brother of one of the attackers in the club.

(ii) How many different teams can be formed which include exactly one of the two brothers? [3]

The two brothers leave the club. The club manager decides that one of the remaining midfielders can play as either a midfielder or as a defender.

(iii) How many different teams can now be formed by the club? [3]

7 Yan is carrying out an experiment with a fair 6-sided die and a biased 6-sided die, each numbered from 1 to 6.

(i) Yan rolls the fair die 10 times. Find the probability that it shows a 6 exactly 3 times. [1]

(ii) Yan now rolls the fair die 60 times. Use a suitable approximate distribution, which should be stated, to find the probability that the die shows a 6 between 5 and 8 times, inclusive. [3]

The probability that the biased die shows a 6 is $\frac{1}{15}$.

(iii) Yan rolls the biased die 60 times. Use a suitable approximate distribution, which should be stated, to find the probability that the biased die shows a 6 between 5 and 8 times, inclusive. [3]

8 **(a)** Sketch a scatter diagram that might be expected when x and y are related approximately by $y = px^2 + t$ in each of the cases **(i)** and **(ii)** below. In each case your diagram should include 6 points, approximately equally spaced with respect to x, and with all x-values positive.

(i) p and t are both positive.

(ii) p is negative and t is positive. [2]

(b) The ages in months (m) and prices in dollars (P) of a random sample of ten used cars of a certain model are given in the table.

m	11	20	28	36	40	47	58	62	68	75
P	112 800	102 600	76 500	72 000	72 000	69 000	65 800	57 000	50 600	47 600

It is thought that the price after m months can be modelled by one of the formulae

$$P = am + b, \qquad P = c\ln m + d,$$

where a, b, c and d are constants.

(i) Find, correct to 4 decimal places, the value of the product moment correlation coefficient between

(A) m and P,

(B) $\ln m$ and P. [2]

(ii) Explain which of $P = am + b$ and $P = c\ln m + d$ is the better model and find the equation of a suitable regression line for this model. [3]

(iii) Use the equation of your regression line to estimate the price of a car that is 50 months old. [1]

9 The number of minutes that the 0815 bus arrives late at my local bus stop has a normal distribution; the mean number of minutes the bus is late has been 4.3. A new company takes over the service, claiming that punctuality will be improved. After the new company takes over, a random sample of 10 days is taken and the number of minutes that the bus is late is recorded. The sample mean is $\bar{t}$ minutes and the sample variance is k^2 minutes^2. A test is to be carried out at the 10% level of significance to determine whether the mean number of minutes late has been reduced.

(i) State appropriate hypotheses for the test, defining any symbols that you use. [2]

(ii) Given that $k^2 = 3.2$, find the set of values of $\bar{t}$ for which the result of the test would be that the null hypothesis is not rejected. [4]

(iii) Given instead that $\bar{t} = 4.0$, find the set of values of k^2 for which the result of the test would be to reject the null hypothesis. [3]

10 A game has three sets of ten symbols, and one symbol from each set is randomly chosen to be displayed on each turn. The symbols are as follows.

Set 1	+	+	+	+	×	×	×	o	o	★
Set 2	+	+	+	×	o	o	o	o	★	★
Set 3	+	+	×	×	×	×	o	o	o	★

For example, if a + symbol is chosen from set 1, a o symbol is chosen from set 2 and a ★ symbol is chosen from set 3, the display would be + o ★.

(i) Find the probability that, on one turn,

(a) ★★★ is displayed, [1]

(b) at least one ★ symbol is displayed, [2]

(c) two × symbols and one + symbol are displayed, in any order. [3]

(ii) Given that exactly one of the symbols displayed is ★, find the probability that the other two symbols are + and o. [4]

11 An art dealer sells both original paintings and prints. (Prints are copies of paintings.) It is to be assumed that his sales of originals per week can be modelled by the distribution Po(2) and his sales of prints per week can be modelled by the independent distribution Po(11).

(i) Find the probability that, in a randomly chosen week,

(a) the art dealer sells more than 8 prints, [2]

(b) the art dealer sells a total of fewer than 15 prints and originals combined. [2]

(ii) The probability that the art dealer sells fewer than 3 originals in a period of n weeks is less than 0.01. Express this information as an inequality in n, and hence find the smallest possible integer value of n. [5]

(iii) Using a suitable approximation, which should be stated, find the probability that the art dealer sells more than 550 prints in a year (52 weeks). [3]

(iv) Give two reasons in context why the assumptions made at the start of this question may not be valid. [2]