# ADDITIONAL MATHEMATICS

**Paper 1**

**4047/01**

**October/November 2015**

**2 hours**

Answer **all** the questions.

1 The function f is defined, for all values of $x$, by

$$f(x) = x^2(1 - x).$$

Find the values of $x$ for which f is an increasing function. [3]

2 The graph of $y = \log_a x$ passes through the points with coordinates (8, 3), (1, $b$) and ($c$, −2).

**(i)** Determine the value of each of the constants $a$, $b$ and $c$. [3]

**(ii)** Sketch the graph of $y = \log_a x$. [2]

3 The number of bacteria in a culture doubles every 3 hours. It is given that $N_0$ is the number of bacteria present at a particular time and that $N$ is the number of bacteria present $t$ hours later. Calculate the value of the constant $k$ in the relationship $N = N_0 e^{kt}$. [4]

4 **(i)** Given that $ax^2 + 6x + c$ is always negative, what conditions must apply to the constants $a$ and $c$? [3]

**(ii)** Give an example of values of $a$ and $c$ which satisfy the conditions found in part (i). [2]

5 **(i)** Sketch the parabola $y^2 = 4x$. [2]

The line $y = x - 1$ intersects the parabola at the points $A$ and $B$.

**(ii)** Show that the mid-point of $AB$ lies on the line $x + y = 5$. [5]

6 Given that $f(x) = 4\cos^2 x - 2\sin^2 x$,

**(i)** express f($x$) in the form $a\cos 2x + b$, stating the value of each of the integers $a$ and $b$, [3]

**(ii)** state the greatest and least values of f($x$), [2]

**(iii)** state the period and amplitude of f($x$). [2]

7 (i) Sketch the graph of $y = |2x - 4|$. [2]

A line of gradient $m$ passes through the point $(0, -1)$.

(ii) In the case where $m = 3$, find the coordinates of any point of intersection of the line and the graph of $y = |2x - 4|$. [3]

(iii) Determine the set of values of $m$ for which the line intersects the graph of $y = |2x - 4|$ in two points. [2]

8 (i) Show that $4\tan\theta + 2\cot\theta = 5\sec\theta$ can be expressed as $2\sin^2\theta - 5\sin\theta + 2 = 0$. [3]

(ii) Hence solve the equation $4\tan 2x + 2\cot 2x = 5\sec 2x$ for $0° < x < 360°$. [4]

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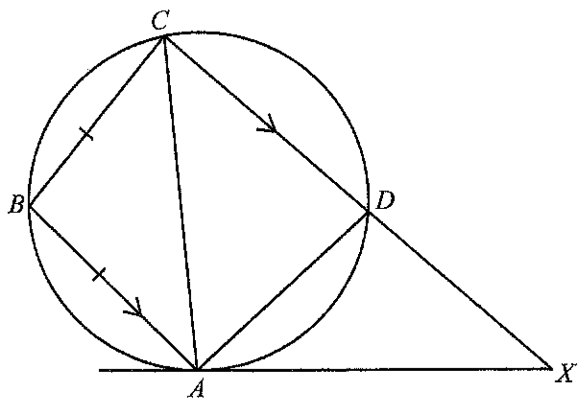


In the diagram, $ABCD$ is a cyclic quadrilateral in which $BA = BC$ and $BA$ is parallel to $CD$. The tangent to the circle at $A$ meets $CD$ produced at $X$.

(i) Show that angle $DAX$ = angle $BCA$. [4]

(ii) Show that triangle $ADX$ is isosceles. [3]

10 A train travelling on a straight track passes a signal $X$ with speed 30 m/s and, 20 seconds later, passes another signal $Y$ with speed 10 m/s. During the journey from $X$ to $Y$ the acceleration, $a$ m/s$^2$, of the train is given by $a = kt - 2$, where $k$ is a constant and $t$ seconds is the time after passing $X$.

(i) Show that $k = 0.1$. [5]

(ii) Find the distance between the signals $X$ and $Y$. [4]



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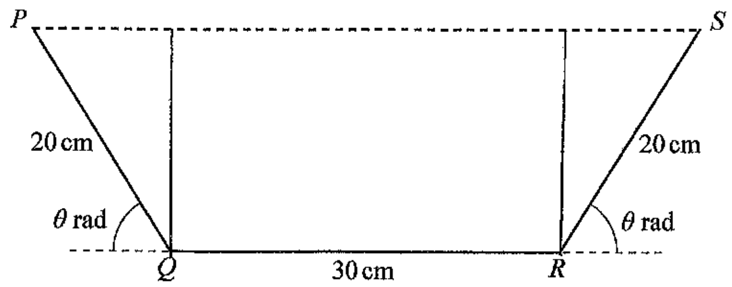



The diagram shows the vertical cross-section $PQRS$ of an open trough made from plastic sheeting. The lengths of $PQ$, $QR$ and $RS$ are 20 cm, 30 cm and 20 cm respectively. The trough rests with $QR$ on horizontal ground and both $PQ$ and $RS$ are inclined at an angle $\theta$ radians to the ground.

(i) Show that the area, $A$ cm$^2$, of the cross-section $PQRS$ is given by

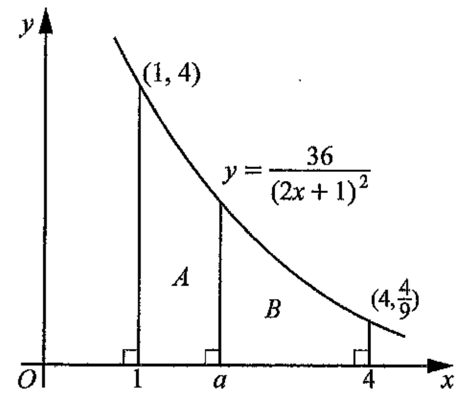
$$A = 600\sin\theta + 200\sin 2\theta.$$ [4]

(ii) Given that $\theta$ can vary, find the value of $\theta$ for which the trough can hold a maximum amount of water. [5]

12 The equation of a curve is $y = \dfrac{36}{(2x+1)^2}$.

(i) A point $P$ moves along the curve in such a way that the $x$-coordinate of $P$ increases at a constant rate of 0.02 units per second. Find the $x$-coordinate of $P$ at the instant the $y$-coordinate is decreasing at a rate of 0.36 units per second. [5]

(ii) The diagram shows part of the curve $y = \dfrac{36}{(2x+1)^2}$ passing through the points with coordinates $(1, 4)$ and $(4, \frac{4}{9})$. Also shown are lines perpendicular to the $x$-axis at the points with $x$-coordinates 1, $a$ and 4.




Given that the areas of the regions marked $A$ and $B$ are equal, find the value of the constant $a$. [5]