Introduction to algebraic language

- Arithmetics deals with computation with numbers.

  Algebra deals with computation with symbols.

- Algebra is very old. Greeks and Babylonians already made algebraic reasoning and computation.

  Middle Age mathematicians refer to the unknown (our x) as "the thing".

- Along 16th century modern notation is introduced.

Introduction to algebraic language

* Makes possible to state general properties:

  For every a and b, it holds $a + b = b + a$.

* Makes possible to reason about unknown quantities, stablishing relationship between them:

  John has participated in a game. He was asked 40 questions. The prize was 150 euros for each correct answer, but there was a penalty of 60 euros for each wrong answer. It was compulsory to give an answer to all the questions. At the end of the game, John got 4530 euros. How many of his answers were correct?

* Algebra allows us to manipulate expressions like previous one (equations) and find their solutions.

Algebra - Arithmetic

* Lucy opened her moneybox and spent half of the money buying a book. After that, she bought an ice cream that cost 2 euros. If she had 7 euros left, how much money was there on the moneybox?

* Solving problems without algebra is important to improve arithmetic understanding. I will ask for that frequently.

* If you have to argue (or give a proof) about some property or fact, it is useful to consider two levels of reasoning.

  Example: We all know that when we add two even numbers we get an even number.

  1. Write an argument that shows that property and that can be presented to a 7 year old kid.
  2. Write a proof using algebraic language..

Exercises

1. Write three “generic” consecutive even numbers.
2. Using the previous expression, show that the sum of three consecutive even numbers is always a multiple of 3.
3. Being even is not important: Write now three “generic” consecutive multiples of 17 and show that their sum is always a multiple of 3.
4. If an even number is multiplied by any integer, the result is always even.
   a) Write an argument that shows that property and that can be presented to a 8 year old kid.
   b) Write a proof using algebraic language..

Algebraic language and patterns

* Look for a pattern:
  - $1+3=4$
  - $1+3+5=9$
  - $1+3+5+7=16$
  - $1+3+5+7+9=25$
* Express it using usual language
* Express it using algebraic language

Algebraic language and patterns

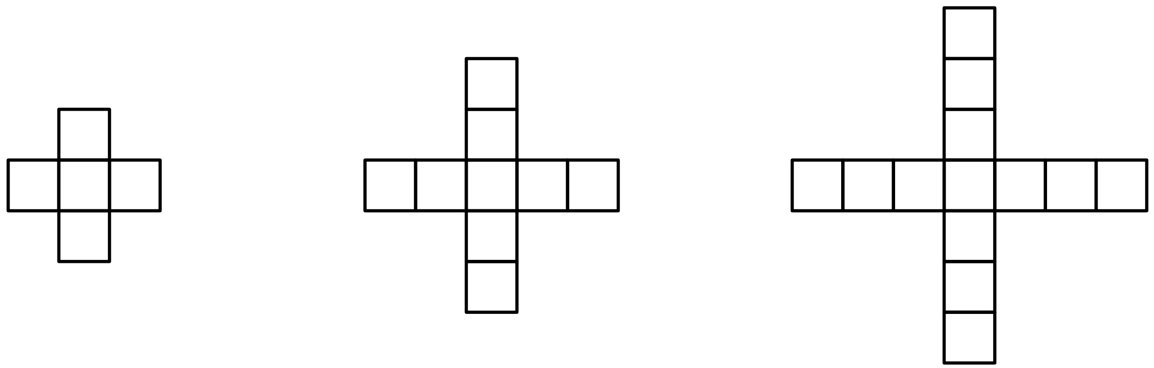
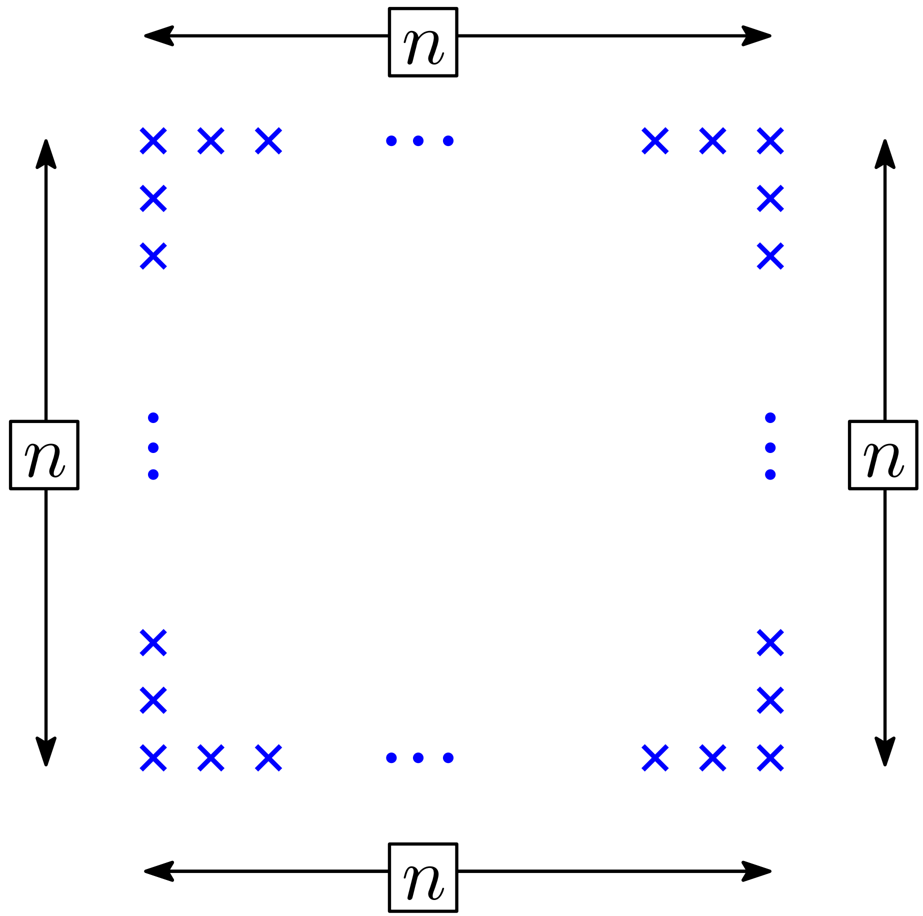


Figure 1 Figure 2 Figure 3

1. How many squares has Figure 8?
2. How many squares has Figure n?

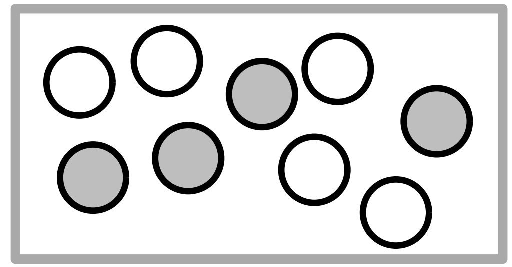
Algebraic language and patterns

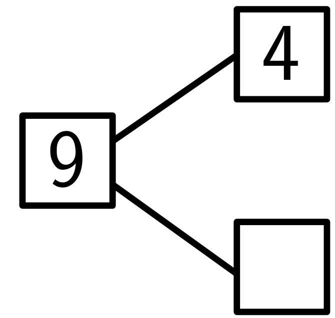


* Think about several ways of counting the crossings in the figure and write the algebraic expression corresponding to each way of counting.

During first years of Primary school

* $\square + 3 = 5$
* $4 = 9 - \square$





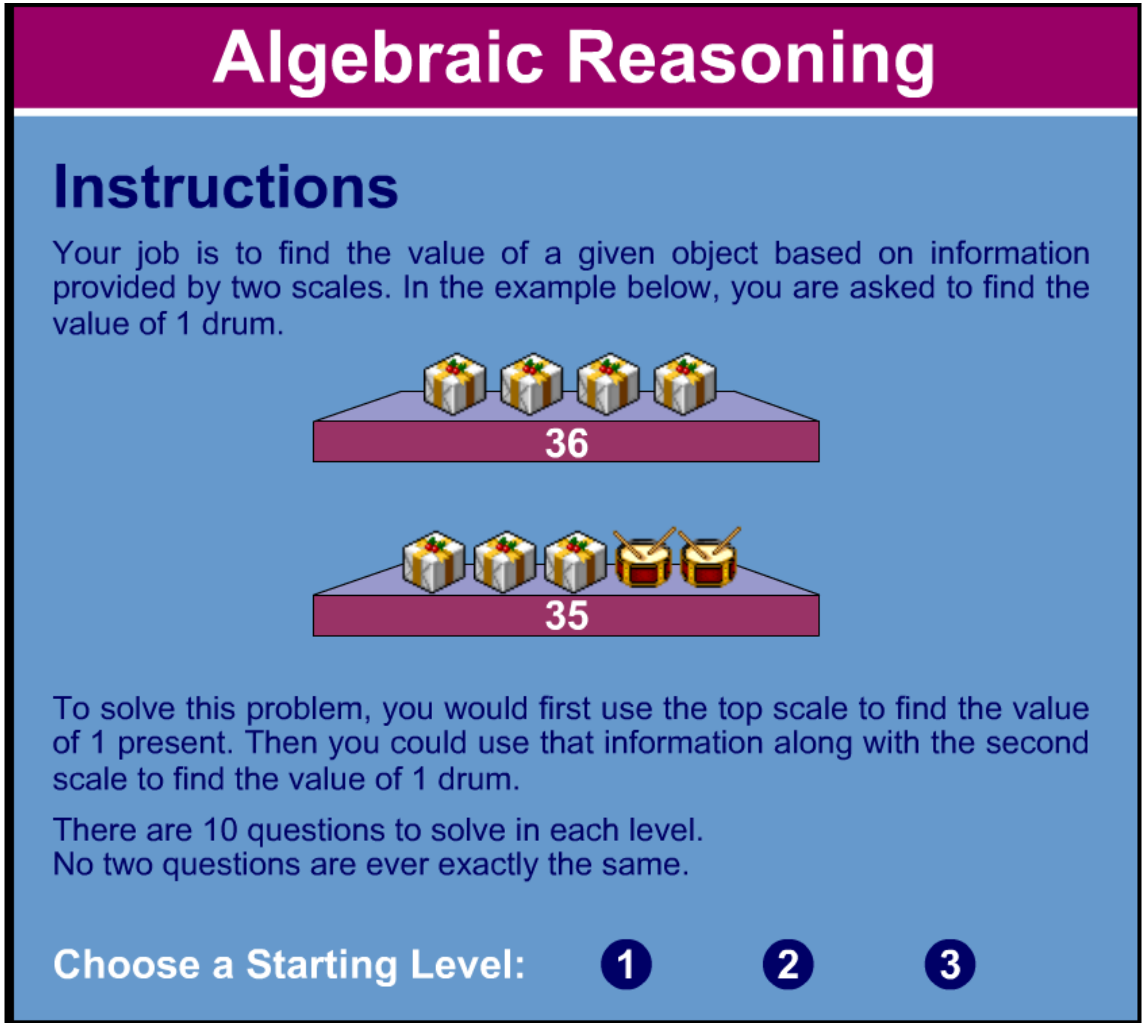
Number bonds

* Things can get more complicated gradually:

$$2 \times \square + 3 = 11$$

Algebraic reasoning

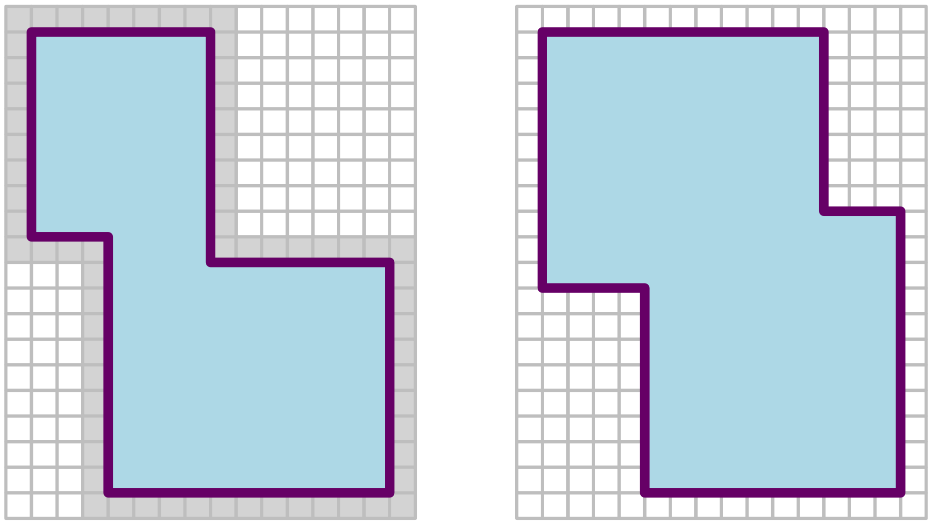
* Algebraic reasoning can be used in a lot of contexts.



http://www.mathplayground.com/algebraic_reasoning.html

Problems

- We want to tile the floor around the swimming pools of the picture, as in the left example. How many tiles will we need?



- Repeat the problem for similar shapes, if the dimensions of the rectangle are a and b.

Problems

* We have a cube, similar to the Rubik cube, made up with $3 \times 3 \times 3$ equal smaller cubes. We paint the exterior faces of the big cube and then we disassemble it.
  1. How many small cubes have 3 painted faces?
  2. How many small cubes have 2 painted faces?
  3. How many small cubes have 1 painted faces?
  4. ¿Cuántos cubos no tienen ninguna cara pintada?

* Repeat the problem with a cube made up with $10 \times 10 \times 10$ small cubes and with a cube made up with $n \times n \times n$ small cubes.