

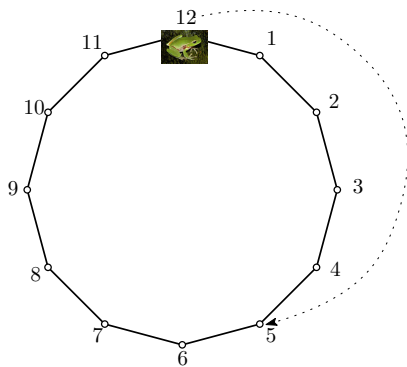
Practice 1.5 (October 24)

Link to the poll: <https://goo.gl/forms/CDXubqHnFvwBynnj1>. Closed: Monday 23, 10 pm.

1. Find all common divisors of 3300 and 1170.
2. Find all pairs of numbers (a, b) such that $\gcd(a, b) = 63$, $\text{lcm}(a, b) = 378$.
3. In the boundary of a trapezoidal field with sides measuring 72, 96, 120, and 132 m we have planted equally spaced trees. Compute the number of planted trees, if we know that there is one in each vertex and that the distance between consecutive trees is as big as possible.
4. Take a number (with at least 3 digits). Add its digits and now subtract that sum from the original number. Check that the number you have obtained is divisible by 9.

Can you explain why?

5. Find all numbers that can be expressed as $87x821y$ that have remainder 3 when they are divided by 5 and remainder 1 when they are divided by 6.
6. Find the values of X and Y such that the number $n = 24X9485Y7$ has remainder 1 when it is divided by 8 and remainder 2 when it is divided by 9.
7. Consider the decimal numbers: $a = 2.\overline{123456123459}$, $b = 3.\overline{12345671234569}$
 - a) Which digit appears in the decimal position 500-th of number a ?
 - b) If we compare the first 1000 digits of both numbers, how many times does digit 9 appear in the same position in both expressions?
 - c) If we start comparing from the decimal position 6000, when will digit 9 coincide again in both expressions?
8. We have a container in the shape of a cuboid with length 6,6 m, width 6,3 m and height 2,52 m. We want to fill it with equal hexahedra in such a way that no gap is left. If the dimensions of the hexahedra have to be an integer (when expressed in centimeters), which measures are possible for the hexahedra?
9. The frog in the drawing starts jumping 5 positions in the dodecagon (in the drawing the first jump is shown). If she continues jumping in the same direction,
 - a) after how many jumps will she end in the initial position for the first time?
 - b) after how many turns will she end in the initial position for the first time?



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10. Given a natural number n , n factorial, denoted by $n!$, is defined in the following way: $n! = n \cdot (n - 1) \cdot \dots \cdot 2 \cdot 1$. For example, $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$.
- a) How many zeros are there at the end of $30!$?
 - b) How many zeros are there at the end of $100!$?
11. Show that when a prime number is divided by 6 the remainder is always 1 or 5.