Lesson 2-2: Decimal numbers

* Decimal numbers with finite expression: decimal fractions.

  A decimal fraction is a fraction that is equivalent to another one whose denominator is a power of 10.

* Example: $3/4$ is a decimal fraction because it is equivalent to $75/100$.

* In 1585 Belgian mathematician Simon Stevin proposed to represent quantities smaller than the unit considering divisions in tenths, hundredths, ... For instance:

$$237 + \frac{4}{10} + \frac{5}{100} + \frac{7}{1000}$$

Decimal numbers

* Decimal part can be seen as generalizing to negative powers of 10 what we have make for the integer part.

$$32 + \frac{4}{10} + \frac{5}{100} + \frac{7}{1000} = 32,457$$

where

$$32,457 = 3 \times 10^1 + 2 \times 10^0 + 4 \times 10^{-1} + 5 \times 10^{-2} + 7 \times 10^{-3}$$

* Review of basic arithmetic with decimal numbers.

1. Standard addition and substraction algorithms are analogous to standard algorithms for integers.

2. Division: $17,3 \div 4 \qquad 17,3 \div 0,4$

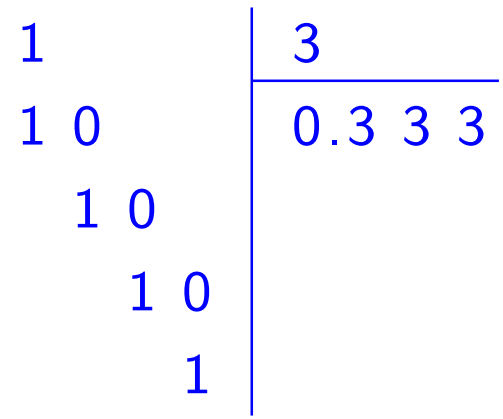
3. Multiplication: $7,3 \times 0,4$

Non-decimal fractions

* How can we express as a decimal number a fraction like $1/3$?

$$\frac{1}{3} = \frac{3}{10} + \frac{1}{30}$$
$$= \frac{3}{10} + \frac{3}{100} + \frac{1}{300}$$
$$= \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{1}{3000}$$

* Exercise: Think about the relation between this and the traditional algorithm

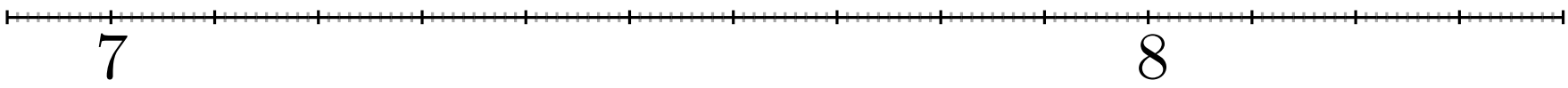


* The fraction $1/3$ has an infinite (and periodic) decimal part: $\frac{1}{3} = 0,3333\cdots = 0,\bar{3}$

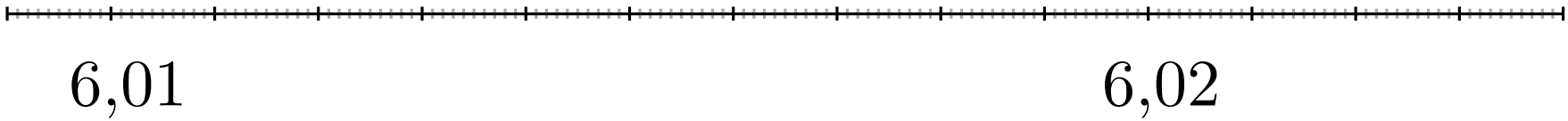
* Mental calculation can be instructive also with decimal numbers. For instance:

a) $2{,}3 \div 0,1$ b) $4 \div 0,2$ c) $27 \times 0,01$

Decimals in the number line



7,08 ? 7,93 ?



6,0105 ? 6,0198 ?

Decimal $\leftrightarrow$ Fraction

* For decimal fractions (decimal numbers with finite expression) conversion is easy.

  Examples:

$$\text{a)} \quad \frac{3}{80} = \qquad\qquad \text{b)} \quad 2{,}87302 =$$

* How can we check whether a given fraction is decimal?

Decimal $\leftrightarrow$ fraction. Infinite expressions

* A lot of rational numbers do not have a finite decimal expression.

  Examples: $1/3$, $2/7$, $4/9$, ...

* A number like $0'333\cdots$ is called repeating decimal number, and it is denoted by $0.\overline{3}$.

* In a repeating decimal number there are two possibilities:

  1. The full decimal part is repeated, as in

     $0.\overline{376} = 0{,}376376376\cdots$ pure repeating decimal
  2. Only a part of the decimal part is repeated, as in

     $0{,}405\overline{37} = 0'405373737\cdots$ mixed repeating decimal

Decimal expression of rational numbers

* Theorem: The decimal expression of every rational number is either finite or repeating (pure or mixed).

* Examples:

$$\frac{1}{6} = 0{,}1\overline{6} \quad \frac{1}{7} = 0{,}\overline{142857} \quad \frac{2}{17} = 0{,}\overline{1176470588235294}$$

* How to express a repeating decimal as a fraction: "fracción generatriz".

* Example: express as an irreducible fraction

a) $2{,}\overline{7}$ b) $2{,}3\overline{75}$

Final remarks

* Decimal expression is not unique.

  a) $0{,}23 = 0{,}23000$

  b) $1 = 0{,}9999\cdots = 0.\overline{9}$

  c) $0'23 = 0'22\overline{9}$

* The set of decimal numbers bigger than zero and smaller than one cannot be listed. (It is not countable).